Coasian Bargaining in World Politics: Re-Examining the Demand for International Regimes

Appendix A: Proofs

Proposition 1

In Fearon (1995).

Proposition 2

Proof. When making an offer C solves

$$\underset{x,T}{Max}\ T - c(x)$$

subject to

$$v(x) - T \geq w_D$$
$$T - c(x) \geq w_C$$

In any equilibrium, C's optimal offer is such that D is indifferent between accepting and going to war. Thus the first constraint binds. Since war is inefficient and payoffs to war are $w_C + w_D$ it must be that C's equilibrium payoff strictly exceeds its war payoff. Thus we can ignore the second constraint. C problem can now be written as the following (unconstrained) maximization problem,

$$\underset{x}{Max}\ v(x) - w_D - c(x)$$

The optimal x is given by the first order condition,

$$-c'(x^*) + v'(x^*) = 0$$

So the efficient division of x is always chosen. Substituting x^* into the first constraint, we obtain an expression for optimal transfers,

$$T^* = v(x^*) - w_D$$

□

Proposition 3

Proof. Let $i \in \{C, D\}$ be chosen as proposer and label whichever player is not chosen j. By Proposition 2, j's equilibrium (*ex post*) utility is w_j. Proposer i's *ex post* utility can be written as joint utility less the utility of player j,

$$\begin{aligned} U_i^{\text{ex post}} &= v(x^*) - c(x^*) - w_j \\ &= s(x^*) + w_i \end{aligned}$$

Ex ante utility for $i \in \{C, D\}$ is then,

$$\begin{aligned} U_i &= \mu(s(x^*) + w_i) + (1 - \mu)w_i \\ &= w_i + \mu s(x^*) \end{aligned}$$

□

Proposition 4

Proof. By optimality a Defender of type w will accept an offer (x', T') if and only if,

$$v(x') - T' \geq w$$

Denote by w' the defender type who is indifferent between accepting or rejecting the equilibrium offer. Also note that if type w' optimally accepts an offer then so do all defenders with types, $w \in (0, w')$, while all others reject.The Challenger's problem is,

$$\underset{x,T}{Max} \int \left[\mathbb{1}(v(x') - T' \geq w)(T - c(x)) + \mathbb{1}(w \geq v(x') - T')w_C\right] dF(w) =$$

$$\underset{x,T}{Max} \int_0^{w'} (T - c(x))dF(w) + \int_{w'}^{\infty} w_C dF(w)$$

Recall from Proposition 2 that for any w' the Challenger's optimal offer is $(x^*, v(x^*) - w')$ with resulting payoff $U_C = v(x^*) - c(x^*) - w'$. We can re-write the Challenger's problem,

$$\underset{w'}{Max} \quad F(w')(v(x^*) - w' - c(x^*)) + (1 - F(w'))w_C$$

Re-arranging the first order condition gives an expression for the optimal w' which we denote w^*,

$$\frac{F(w^*)}{f(w^*)} = [v(x^*) - c(x^*) - w^* - w_C] \tag{1}$$

This is well-defined given the assumptions on f and A1. It is also unique since the assumptions on f imply $\frac{dF(x)/f(x)}{dx} > 0$.[1] The implied equilibrium offer is $(x^*, v(x^*) - w^*)$ which is accepted by all types $w \leq w^*$ and rejected by all $w > w^*$. □

Proposition 5

Proof. Recall from Proposition 1 that in the absence of side payments the Defender will accept an offer if and only if $v(x) \geq w_D$. Again, denote the type which is indifferent between accepting and rejecting by w'. By a similar logic to the last result the Challenger's problem is,

$$\underset{w'}{Max} \quad F(w')[-c(x(w'))] + (1 - F(w'))w_C$$

where $x(w') = v^{-1}(w')$ and is well-defined by the assumptions on $v(\cdot)$.[2] Denote by w^r the optimal indifferent type and by x^r the implied policy proposal.[3] Re-arranging the first order

[1] For discussion see Little and Zeitzoff (2017, p.550).

[2] The optimal policy proposal, x, depends here on the indifferent type, w'. In contrast, above the optimal offer is always the efficient policy, x^*, simplifying the analysis in that case.

[3] That is, $x^r = v^{-1}(w^r)$.

condition, we have

$$\frac{F(w^r)}{f(w^r)} = \frac{v'(x^r)}{c'(x^r)}\big[-c(x^r) - w_c\big] \tag{2}$$

Optimality for the Challenger as well as the assumptions on v and c ensure that the right hand side is strictly positive. As noted above the assumptions on f ensure that $\frac{dF(x)/f(x)}{dx} > 0$ so that the solution is unique. Moreover this implies $w^* \geq w^r$ if and only if,

$$\frac{F(w^*)}{f(w^*)} \geq \frac{F(w^r)}{f(w^r)}$$

or, by equations (1) and (2),

$$\frac{v(x^*) - c(x^*) - w^* - w_C}{-c(x^r) - w_C} \geq \frac{v'(x^r)}{c'(x^r)}$$

where the denominator on the left hand side must be strictly positive due to the Challenger's participation constraint and the left hand side overall must be greater than one due to the efficiency of x^*.

□

Proposition 6

Proof. The challengers sets a schedule $[x,\ T(x)]$. Let x_i represent the amount of x received by type i. In the always peaceful solution, the challenger's problem is

$$\begin{array}{rcr} & \max_{T(x)} \beta\left[T(x_L) - c(x_L)\right] + (1-\beta)\left[T(x_H) - c(x_H)\right] & \\ \text{subject to} & x_i = \arg\max_x v_i(x) - T(x) & \text{for } i = L,\ H \\ \text{and} & v_i(x_i) - T(x_i) \geq w_D & \text{for } i = L,\ H \end{array}$$

That is, there are four constraints. First, each type of defender must prefer to select the contract intended for him rather than the contract intended for the other type (incentive compatibility). Second, each type must prefer to select the contract intended for him rather than reject both offers and risk war (individual rationality).

Applying the revelation principle, we can restrict each schedule $T(x)$ to the pair of optimal choices made by each type of defender so $\{[T(x_L),\ x_L]$ and $[T(x_H),\ x_H]\}$ and define

$T(x_i) = T_i$. The above problem can now be rewritten as

$$\max_{T_i,\, x_i} \beta \left[T_L - c(x_L)\right] + (1-\beta)\left[T_H - c(x_H)\right]$$

subject to

$$\begin{array}{ll} IC_H & v_H(x_H) - T_H \geq v_H(x_L) - T_L \\ IC_L & v_L(x_L) - T_L \geq v_L(x_H) - T_H \\ IR_H & v_H(x_H) - T_H \geq w_D \\ IR_L & v_L(x_L) - T_L \geq w_D \end{array}$$

Next, note that if $v'_H > v'_L$, then IC_H and IR_L jointly imply IR_H,

$$v_H(x_H) - T_H \geq v_H(x_L) - T_L > v_L(x_L) - T_L \geq w_D$$

Next, for now omit the constraint IC_L. This simplifies the problem to

$$\max_{T_i,\, x_i} \beta \left[T_L - c(x_L)\right] + (1-\beta)\left[T_H - c(x_H)\right]$$

subject to

$$\begin{array}{ll} IC_H & v_H(x_H) - T_H \geq v_H(x_L) - T_L \\ IR_L & v_L(x_L) - T_L \geq w_D \end{array}$$

In this problem, both constraints must bind. IC_H must bind since otherwise the challenger could raise T_H until it did, improving her own payoff without affecting either type's incentives. IR_L must also bind since otherwise the challenger could do better raising T_L until it does. Doing so would relax IC_H, but otherwise leave incentives unchanged. Since both of these constraints bind, we can substitute them directly into the maximand. This yields the following (unconstrained) maximization problem,

$$\max_{x_L,\, x_H} \beta \left[v_L(x_L) - w_D - c(x_L)\right] + (1-\beta)\left[v_H(x_H) - w_D - c(x_H) - v_H(x_L) + v_L(x_L)\right]$$

The first order conditions are,

$$\begin{array}{ll} x_H: & v'_H(x^*_H) = c'(x^*_H) \\ x_L: & \beta[v'_L(x^*_L) - c'(x^*_L)] - (1-\beta)[v'_H(x^*_L) - v'_L(x^*_L)] = 0 \end{array}$$

Re-arranging the second condition we obtain,

$$v_L'(x_L^*) = \beta c'(x_L^*) + (1-\beta)v_H'(x_L^*)$$

Since $\beta \in (0,1)$ and $v_H' > v_L'$ this implies that $v_L'(x_L^*) > c'(x_L^*)$. In turn, by the assumptions on v_i and c we have $x_H^* > x_L^*$. Equilibrium transfers are,

$$\begin{aligned} T_H^* &= v_H(x_H^*) - v_H(x_L^*) + v_L(x_L^*) - w_D \\ T_L^* &= v_L(x_L^*) - w_D \end{aligned}$$

$x_H^* > x_L^*$ then implies $T_H^* > T_L^*$. Finally, note that IC_L, which we ignored above, does in fact hold. Recall that IC_H binds at the optimum,

$$v_H\left(x_H^*\right) - T_H^* = v_H\left(x_L^*\right) - T_L^*$$

Then $T_H^* > T_L^*$ and $x_H^* > x_L^*$ imply,

$$v_L\left(x_L^*\right) - T_L^* > v_L\left(x_H^*\right) - T_H^*$$

Recall from above that if $x_L^* > 0$, then $v_L'\left(x_L^*\right) \neq c'\left(x_L^*\right)$. The peaceful offer to the L type defender is always inefficient.

Note that in order for the challenger to be willing to make both offers, it must be the case that,

$$T_L^* - c(x_L^*) > w_C$$

Otherwise the challenger prefers to make only a single, risky offer which will be accepted by the high valuation type and rejected by the low valuation type. War then occurs on the equilibrium path whenever,

$$w_C > T_L^* - c(x_L^*)$$

When this inequality holds, war occurs with probability β. This inequality will sometimes be satisfied in spite of the inefficiency of war. This is the case precisely because T_L^* and x_L^* are not the optimal values for when D is known to be a L type, but instead the optimal distorted value for dealing with both types. Plugging in for T_L^* in the above, we can write the war condition as,

$$w_C + w_D > v_L(x_L^*) - c(x_L^*)$$

Given the inefficiency of the peaceful offer made to type L we have,

$$U_C + U_D > v_L(x_L^*) - c(x_L^*)$$

where U_C and U_D are the utilities implied by the optimal level of exchange. By assumption A1 war is inefficient, implying $U_C + U_D > w_C + w_D$, though the war values are otherwise unrestricted. So for any w'_C, w'_D such that,

$$U_C + U_D > w'_C + w'_D > v_L(x_L^*) - c(x_L^*)$$

assumption A1 is satisfied, and a risky offer is made leading to war with probability β.

□

Proposition 7

Proof. To explore how shifts in the relative balance of power interact with side payments we can relate outcomes in the revised version of the model with equilibrium behavior in the static game. By Proposition 2 the equilibrium offer in the static model is $(x^*, v(x^*) - w_D)$. Provided that $b > v(x^*) - w_D - \Delta$, the second period payoff to the challenger is,

$$\frac{\delta}{1-\delta}[v(x^*) - c(x^*) - w_D - \Delta]$$

If the bargaining outcome in period one remains as in the static case, the challenger's total payoff is,

$$v(x^*) - c(x^*) - w_D + \frac{\delta}{1-\delta}[v(x^*) - c(x^*) - w_D - \Delta]$$

The challenger prefers peaceful bargaining in both periods to war if and only if,

$$\alpha = \frac{1-\delta}{\delta}[v(x^*) - c(x^*) - w_D - w_C] + v(x^*) - c(x^*) - w_D \geq \Delta$$

Next note that as Δ increases, the first period transfer must increase accordingly to compensate the challenger for worsening terms of future cooperation. If this change is small the efficient level of cooperation, x^*, will remain unchanged as shifts in power are compensated for only via the side payment. At the extreme, the challenger will enjoy the maximum side payment b while continuing to prefer peaceful settlement to war provided,

$$\beta = \frac{1-\delta}{\delta}[b - c(x^*) - w_C] + v(x^*) - c(x^*) - w_D \geq \Delta$$

If the shift in power becomes still more extreme, the challenger will demand not only the maximum first period transfer, but a growing first period concession x. In the most extreme case where the equilibrium offer is $x = 0$, the challenger prefers peaceful bargaining to war provided,

$$\gamma = \frac{1-\delta}{\delta}[b - w_C] + v(x^*) - c(x^*) - w_D \geq \Delta$$

□

Appendix B: Additional Results

Issue Indivisibility

We next turn to the question of issue-linkage and issue indivisibility. In contrast to conventional wisdom we show that for some forms of indivisibility, bargaining failure may occur even in the presence of side payments. First we consider a case in which issue linkage is in fact sufficient to prevent bargaining failure. Suppose that preferences are defined as before but that the set of feasible bargains is restricted to $\mathcal{B} = \{0, \bar{x}\}$. In other words the issue under negotiation is no longer divisible. For our first case we make the following assumption, similar in spirit to Assumption A1 above.

There exists an $x \in \{0, \bar{x}\}$ such that,

$$\textbf{A2}. \quad v(x) - c(x) \geq w_C + w_D$$

Then the following result obtains.

Proposition 1 (Indivisibility with No Bargaining Failure)**.** *Bargaining failure never occurs in the presence of transfers.*

Assumption A2 ensures that at least one of the feasible bargaining outcomes is jointly preferred to the outcome of bargaining failure. In this case, side payments have the expected effect of increasing efficiency and making cooperation possible. Note though that there are many cases in which assumption A2 will not hold. If state preferences are defined over a continuous dimension while the bargaining space is exogenously restricted it may well be the case that any policies jointly preferred to bargaining failure fall outside of the feasible set.

As an example consider the decision to establish a multilateral regime for international finance. In this case states may hold differing preferences over the underlying dimension of financial volatility, and there may exist a level of volatility that would improve the welfare of all if it were attainable under a multilateral regime. Yet given the technocratic challenges of institutional design and the uncertainty of how various actors will respond to any new regime it may in practice be impossible to achieve the Pareto improving outcome. Instead states confront a finite number of proposed institutional arrangements, implying indivisibilities in the issue under negotiation but with no guarantee that the set of feasible outcomes contains one which is jointly welfare enhancing.

Relaxing assumption A2 while maintaining instead assumption A1, Proposition 2 establishes that issue-linkage may well be insufficient to prevent bargaining failure.

Proposition 2 (Indivisibility with Bargaining Failure)**.** *Bargaining failure may occur in the presence of transfers.*

It should be noted that, as in the example above, bargaining failure here reflects inefficiencies arising from the cooperative technology rather than from any feature of the bargaining protocol itself. Nonetheless this type of inefficiency is likely to be particularly common in international cooperation, where cooperation commonly requires complex and unpredictable institutional arrangements. Moreover in nearly all bargaining contexts, state preferences reflect the aggregation of the preferences of various societal actors or coalitions. Of course where this is the case there is no guarantee that state preferences will meet the requirements of rationality. Yet even where aggregate preferences remain well-defined they are likely to exhibit discontinuities which may similarly - though artificially - restrict the set of feasible international bargains.[4]

Proofs of Additional Results

Proposition 1

Proof. If the challenger offers (x, T), where $x \in \{0, \bar{x}\}$, the defender will accept if and only if,

$$v(x) - T \geq w_D$$

In any equilibrium this must hold with equality. If the challenger offers $\bar{x}$ the optimal transfer is then $T^{\bar{x}} = v(\bar{x}) - w_D$ while if the challenger offers 0 the optimal transfer is $T^0 = v(0) - w_D$. The challenger's equilibrium utility from cooperation is then,

$$\max_{x \in \{0, \bar{x}\}} v(x) - c(x) - w_D$$

No bargain is possible if the challenger derives greater utility from bargaining failure than from cooperation,

$$w_C > \max_{x \in \{0, \bar{x}\}} v(x) - c(x) - w_D$$

[4] As Fearon (1995) notes "international issues may often be *effectively* indivisible, but the cause of this indivisibility lies in the domestic political and other mechanisms rather than in the nature of the issues themselves" (382).

But this cannot be by assumption $A2$. Thus there always exists an offer which C prefers to make and D prefers to accept. $\square$

Proposition 2

Proof. By assumption A1, there exists an $x^* \in [0, \bar{x}]$ such that,

$$v(x^*) - c(x^*) > w_C + w_D$$

It may still be the case that,

$$w_C > \max_{x \in \{0, \bar{x}\}} v(x) - c(x) - w_D$$

so that there is no offer that both challenger and defender prefer to bargaining failure.

$\square$

References

James D. Fearon. Rationalist explanations for war. *International Organization*, 49(3):379–414, 1995.

Andrew T. Little and Thomas Zeitzoff. A bargaining theory of conflict with evolutionary preferences. *International Organization*, 71:523–557, 2017.