Reasoning about War with Uncertainty about Victory

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Abstract

We analyze a crisis interaction between two states in which each side has private information about their military capabilities and these capabilities jointly determine the probability that either state will win a war. Each country can choose to proceed to a negotiated settlement with an uncertain value, or to start a war to try and acquire the object of dispute by force. We explore how the decision-makers' beliefs about the cost of war and probability of victory are related to the decision to fight or settle. We show that when either side can unilaterally start a war, the interdependent nature of uncertainty about the probability of victory can lead decision-makers to start wars in a wide variety of circumstances. We find that wars can start when both sides are "pessimistic" about their chances of victory, that mutual optimism is never necessary for war and, sometimes, mutual optimism is not even sufficient. This is true for both rational decision-makers and those who have bounded rationality in their learning process.

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1 Introduction

The question of why states fight costly wars when Pareto efficient peaceful settlements are available is central to understanding the causes of war. One argument that has received attention in the scholarly and policy literatures is associated with "mutual optimism" on the eve of conflict (Blainey 1988, Wagner 1994, Kim & Bueno de Mesquita 1995, Van Evera 1999). As clearly articulated by Blainey (1988), mutual optimism causes war when two countries have private estimates of their ability to prevail in a war and these estimates preclude them from accepting a peaceful settlement of the dispute. More specifically, if both countries are optimistic about their prospects in war—perhaps because both sides believe they are more likely to prevail than lose—then no peaceful settlement may satisfy both sides. With these optimistic beliefs about the chances of victory, one side or the other rejects any peaceful settlement, and costly conflict follows. As a consequence, the argument goes, wars often occur when both sides are optimistic about their chances of reaching their goals through fighting a war.

In considering what causes war, one difficulty that arises is that there are, in fact, many possible causes of war. Fearon (1995), for example, describes three rationalist explanations for war. In order to concentrate on a specific cause of war, then, we must somehow distinguish it from other possible causes. Given this, how might the the effect of beliefs about the likelihood of victory be studied? One answer is to create a general model in which mutual optimism should produce war and, conversely, a lack of mutual optimism should produce peace. If war occurs in these environments, the mutual optimism explanation for war gains theoretical credence. If war does not occur in this then, at a minimum, both sides being optimistic about success in war is one of a set of ingredients that lead to war. Fey & Ramsay (2007) show that in environments where countries must agree to forego a settlement for war to occur, and therefore mutual optimism is necessary for war, war does not occur. In Fey & Ramsay (2016), they show that mutual optimism is never necessary and sufficient, and sometimes is neither, in the bargaining model of war with private information about military capabilities, under a variety of definitions of mutual optimism.

In this paper, we consider another way of thinking about uncertainty about victory as an explanation of war. We move away from environments in which both sides must agree to fight, as in Fey & Ramsay (2007) or only one side has the option to fight, as in the standard bargaining model in which war occurs only by the choice of the second side to reject the proposer's offer (Yildiz 2011, Slantchev & Tarar 2011, Fey & Ramsay 2016).[1] Instead, we focus on situations where a "unilateral war" assumption permits either side to start a war if it is dissatisfied with the peaceful alternative. Importantly, in our analysis both sides have uncertainty about the strength of the other side, which ensures that mutual optimism is possible. This differs from the bargaining model of Slantchev and Tarar, which only has uncertainty on one side. But it also differs from Fey & Ramsay (2016) in that they abstract away from the settlement process and assume a settlement that is a function of the state rather than an explicit bargaining protocol.[2]. In this analysis there must be mutual agreement to settle.

Our study of environments with uncertainty about the probability of winning leads us to focus on the "strategic inferences" that a rational decision-maker should make about the information of an opponent in an equilibrium theory of war. These inferences do not arise in models with privately valued costs because in such models the value of war to one side does not depend on the private information of the other side. In particular, in such models country 1 does not care which types of country 2 choose war; it only cares about how likely it is that war is chosen. With uncertainty about a commonly valued parameter like the probability of victory in war, however, the value of war depends on the private information of both sides. Therefore, country 1 now cares which types of country 2 choose war, which is something that is determined by the equilibrium strategy of country 2. In this way, country 1 must make a strategic inference about the information of country 2 in choosing its optimal decision. These inferences take a particularly powerful form in games with unilateral war, which are the focus of our study. Specifically, in such a strategic context, if your opponent is choosing to go to war, then your choice does not matter—the outcome is war no matter what. Put another way, your choice matters only if your opponent is choosing to not unilaterally start a war. Thus, in deciding on an optimal choice of action, a decision-maker should condition on the fact that their opponent is not fighting. This, in turn, means that in equilibrium a decision-maker acts in equilibrium as if she has different information than she possesses. In this way, our analysis is related to the winner's curse in auction theory (Thaler 1994) and the swing voter's curse in voting theory (Feddersen & Pesendorfer 1996). In international conflict, as we will show, strategic inferences can, among other things, lead

[1] However, see Leventoğlu & Tarar (2008) for a bargaining model in which the proposer can choose to fight.

[2] Since settlement only occurs when both sides choose not to fight there is no real ability to signal toughness through the decision to settle and therefore this expected value is well defined even if the settlement does not explicitly depend on the action profile.

pessimistic leaders to start wars and optimistic leaders to accept settlements.

The idea of mutual optimism leading to war is also related to work on misperceptions, overconfidence, and war (Conrad & Sanford 1943, Sanford, Conrad & Franck 1946, Tuchman 1962, Jervis 1968, Jervis 1982, Levy 1983, Stein 1982, Jervis 1988, Johnson 2004). This vast literature speaks to many factors influencing perceptions and beliefs. The mutual optimism literature, and our analysis to follow, focuses on actors beliefs about capabilities and, following Holsti (1962), is a model of how decision-makers' beliefs are derived from the available information. Thus our theoretical analysis provides a rigorous benchmark and formal foundation for a theory of misperceptions.

We introduce the main ideas of the analysis in Section 2 by way of two examples. Both examples involve a simple crisis game between two countries in which both sides in turn have the option of fighting to change some fixed status quo. Both sides are uncertain as to the strength of the other side—mutual optimism occurs if they both believe they can do better by war than through settlement. In our first example, we show that there can be states of the world in which mutual optimism is present but war does not occur. That is, mutual optimism need not be sufficient for war. Also in this example, we show that war occurs in some states of the world in which beliefs are optimistic or pessimistic. We build on this in the second example and show that, surprisingly, war can occur even when both sides are pessimistic. That is, we show that even if neither side is unilaterally optimistic, an equilibrium exists in which war occurs.

While these examples are quite suggestive, it is important to consider how general these results may be. To this end, in Sections 3 and 4 we describe a general class of models with unilateral war—either side can reject a peaceful settlement and choose war instead. Our first general result answers the question: what leads to war in these environments? We show that the existence of a type of one country that is unilaterally optimistic about the outcome of a war implies that the overall equilibrium probability of war is positive. So while it is not the case that countries with optimistic expectations regarding war always choose to fight, nor is it the case that countries with pessimistic expectations never fight, a single side with optimistic expectations is sufficient to guarantee a non-zero *ex ante* risk of war in equilibrium. We then turn to considering some additional general results relating mutual optimism to war when any country can unilaterally choose to fight. Our second result is that in every such model, if war occurs in equilibrium, there must be states of the world in which war occurs but mutual optimism is not present. In other words, mutual optimism is never necessary for war. But what if players aren't fully rational? We address this question by showing that in a natural setting where players are limited in their information processing, our general result that mutual optimism is never necessary for war continues to hold.

Summing up these results, we find that unilateral optimism is an important marker for war. While optimistic states may fight or may not in any realized state of the world, the *possibility* of unilateral optimism implies that the *ex ante* probability of war is always positive in any pure strategy equilibrium. Moreover, we show that at any state of the world with mutual optimism, even one in which war does not occur, we can change mutual optimism to unilateral optimism by giving one side full information and the result must be war at that state. Here again, war is strongly associated with the existence of a single optimistic decision-maker.

2 Two Examples

We begin by describing two simple examples that illustrate our main findings. The two examples differ only by what the two sides know about their relative power. The examples share the same game-theoretic structure, with the same actions in the same order. This structure is chosen to be as simple as possible to highlight some important incentives that exist in international conflict. However, as we show in Section 4, our main findings apply to a broad class of models that may more accurately reflect the complexities of international conflict.

2.1 Setup

In both of our examples, we have two countries, labeled 1 and 2, which are involved in an international crisis. This crisis can be resolved peacefully or by the use of force. Although our results apply to a wide variety of game forms, as described in Section 3, in order to keep the examples simple we use as simple a game form as possible. In particular, we suppose country 1 begins by choosing to fight or not fight, represented by the choice of actions F or N. If country 1 chooses to fight, then war results. If country 1 chooses to not fight, then country 2 chooses to fight or not fight. If country 2 chooses to fight, then war results, otherwise the peaceful settlement results. Thus, as illustrated in Figure 1, if *either* country chooses action F, then war results. If both countries choose action N, then the peaceful settlement results. This simple game is a game in which either side can choose to fight. One way to conceptualize this game is that there a status quo allocation that can only be changed by war. Either player can start the war and only if both players accept the status quo does peace prevail.

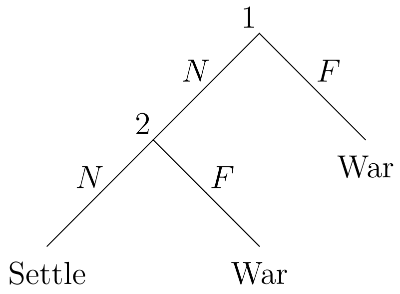

Figure 1: Game Form of Examples

We next describe the payoffs and information structure of our examples. There are two possible outcomes to the game we describe: war and peaceful settlement. For simplicity, we suppose that the peaceful settlement is fixed at a payoff of 1/2 for both countries.[3] Again, this can be viewed as the status quo of equal division of some resource of unit size, for example. In case of war, the outcome is either victory for country 1 and defeat for country 2 or vice versa. We normalize the value of victory to be 1 and the value of defeat to be 0. Regardless of which side wins, war is costly and each side must pay a cost $c_i > 0$ in the event of war. Therefore, if we let p_i denote the probability that country i wins the war, the expected utility for war for country i is given by $p_i - c_i$.

Suppose that each side has private information about its war-fighting ability and that the probability of victory for each side depends on the war-fighting ability of both sides. Let each country be one of three possible types, A, B, and C, and suppose these types are equally likely. We denote that type of country i by $t_i \in \{A, B, C\}$ and thus a type profile $t = (t_1, t_2)$ gives the types of both countries. As is standard, we suppose a country knows its own type but is uncertain about the type of its opponent. We suppose the probability of victory for country i depends on the realized type profile t, which we denote $p_i(t_1, t_2)$. As the probability of winning and therefore the value of war depends on the types of both players, both of our examples have "interdependent values."

Now that we have defined the information structure and payoffs for our game, we next define what it means for a country to be optimistic. Informally, we say that a country

[3] In the general results below, the value of the peaceful settlement can be much more complicated, as it can be an arbitrary function of the state of the world, but the intuition is clearest with the fixed settlement.

	A	B	C
A	(.5,.5)	(.3,.7)	(.9,.1)
B	(.7,.3)	(.5,.5)	(.5,.5)
C	(.1,.9)	(.5,.5)	(.5,.5)

Figure 2: Probabilities of winning: (p_1, p_2)

is optimistic if, based solely on its own private information, it thinks it will be better off fighting a war than receiving the peaceful settlement. Formally, we let $\hat{p}_i(t_i)$ denote the *naive* conditional probability that type t_i of country i will win a war. Thus,

$$\hat{p}_i(t_i) = \frac{1}{3} \sum_{t_j \in \{A,B,C\}} p_i(t_i, t_j),$$

the average probability of victory across a row or down a column. We say type t_i of country i is optimistic if $\hat{p}_i(t_i) - c_i > 1/2$ and we say there is mutual optimism at type profile $t = (t_1, t_2)$ if both type t_1 of country 1 and type t_2 of country 2 are optimistic. It is important to emphasize that these definitions are naive in that they refer to a country's likelihood of victory without conditioning on which types of their opponent would actually choose to fight.

2.2 Example 1: Information and Equilibrium

The exact way in which the probability of victory varies with the type profile in our first example is given by the table in Figure 2. In this table, the type of country 1 corresponds to the rows and the type of country 2 corresponds to the columns.

The values for the probability of winning are chosen to make the presentation of the results in this example as clear as possible. But it is possible to provide some motivation for these values, as follows. Consider an emerging, unproven, new technology for warfare such as poison gas or the tank in World War I or unmanned aerial vehicles (UAVs) in the early 1990s. Now suppose that the two countries in our example have varying abilities in regards to this new technology and these abilities are private information. Specifically, a type A country has the ability to use this new technology in an offensive role, such as having stock piles of poison gas and a reliable delivery system. A type B country has an effective defense against this technology, such as the distribution of gas masks, but no ability for offensive use. Finally, a type C country has neither of these abilities. This interpretation of the types

	A	B	C	
A	(.5,.5)	(.3,.7)	(.9,.1)	Opt.
B	(.7,.3)	(.5,.5)	(.5,.5)	Opt.
C	(.1,.9)	(.5,.5)	(.5,.5)	Pess.
	Opt.	Opt.	Pess.	

Figure 3: Optimism and pessimism ($c_i < 1/15$)

of the countries motivates the values in Figure 2 in the following way. If the two countries are the same type, then neither has an advantage: if both are type A they both have an equal offensive advantage and if both are type B or C then the new technology is not used offensively. If one country is type B and the other is type C, then again neither has an advantage because the new technology is not used offensively. On the other hand, if a type A country fights a type C country, then the type A country has an overwhelming advantage and wins with probability .9. Finally, if a type A country faces a type B country in war, the defensive capability of the type B country eliminates the offensive abilities of type A country which makes victory more likely for the type B country. Of course, this description is only meant to make the values in our example plausible, it is not an attempt to explain war generally.

In our example, it is easy to calculate the *naive* conditional probabilities that country i will win a war:

$$\hat{p}_i(A) = \hat{p}_i(B) = \frac{17}{30} \quad \text{and} \quad \hat{p}_i(C) = \frac{11}{30}.$$

Thus, types A and B of country i are optimistic when $c_i \in (0, 1/15)$ and type C of country i is never optimistic. Intuitively, the requirement that c_i be relatively small to be optimistic should make sense; if war is extremely costly then war will never be a better choice than peace, regardless of a country's private information. We summarize the optimism or pessimism of each type in Figure 3.

We are now ready to identify the equilibria of this game. In fact, when $c_i \in (0, 1/15)$ for $i = 1, 2$, this game has a unique perfect Bayesian equilibrium. In this equilibrium, each country plays action F if its type is A and plays action N if its type is B or C. To see that this strategy profile is indeed an equilibrium, consider country 2. Because of the timing of the game, country 2's choice matters only when country 1 is choosing action N. Given country 1's strategy, this occurs precisely when $t_1 = B$ or $t_1 = C$. Therefore, for type A of

	A	B	C	
A	(.5,.5)	(.3,.7)	(.9,.1)	Opt.
B	(.7,.3)	(.5,.5)	(.5,.5)	Opt.
C	(.1,.9)	(.5,.5)	(.5,.5)	Pess.
	Opt.	Opt.	Pess.	

Figure 4: Type pairs at war and peace

country 2, conditional on its action mattering, its expected payoff for war is

$$E[p_2(t_1, A) \mid t_1 \in \{B, C\}] - c_2 = \frac{.3 + .9}{2} - c_2 = .6 - c_2$$

As $c_2 < .1$, choosing F is superior to choosing N. Similarly, for type B or C of country 2, its expected payoff for war conditional on its action mattering is $(.5 + .5)/2 - c_2 = .5 - c_2$. As $c_2 > 0$, these types of country 2 prefer to choose N rather than F. Turning now to the choice of country 1, note that country 1's choice matters only when country 2 is choosing action N. Given country 2's strategy, this occurs precisely when $t_2 = B$ or $t_2 = C$. Therefore, the analysis for country 1 is exactly symmetric to the analysis just described for country 2.[4] Thus the given strategy profile is an equilibrium.[5]

2.3 Example 1: Implications

The equilibrium in our first example illustrates several important aspects of the connection between optimistic beliefs and war that can be seen in Figure 4. In this figure, the type pairs for which war occurs are shaded and the type pairs for which the peaceful settlement occurs are unshaded.

There are two main observations to make about this example. The first observation is that mutual optimism is not necessary for war. If it were necessary, then it would be the case that mutual optimism is present at every type profile for which there is war. But for both type profiles (A, C) and (C, A), war occurs but mutual optimism is not present, because type C of both countries is not optimistic.

The second observation is that mutual optimism is not sufficient for war. If it were, then it would be the case that war occurs at every type profile at which mutual optimism is

[4] This symmetry in the analysis is why the exact timing of the game in our example is inconsequential.

[5] The proof that this equilibrium is unique is somewhat involved and, therefore, is presented in a Reviewer's Appendix.

	A	B	C	
A	(.5,.5)	(.5,.5)	(.65,.35)	Pess.
B	(.5,.5)	(.5,.5)	(.65,.35)	Pess.
C	(.35,.65)	(.35,.65)	(.5,.5)	Pess.
	Pess.	Pess.	Pess.	

Figure 5: Probabilities of winning and pessimism ($c_i > .05$)

present. But for the type profile (B, B), there is mutual optimism but war does not occur. For both sides, type B has the naive expectation that it will do better in war than in a peaceful settlement, so mutual optimism is present at type profile (B, B). But in the unique perfect Bayesian equilibrium of the game, type B of both countries chooses not to fight. Thus war does not occur at this type profile.

2.4 Example 2: Information and Equilibrium

In our second example, we maintain the game form and general information framework described above. This example differs, though, in how the probability of victory varies with the type profile of the two sides. These probabilities are given by the table in Figure 5.

It is not difficult to motivate the probabilities in this figure. A type C country has some vulnerability that can be exploited by a type A or B country, while those two types of a country have no significant advantage or disadvantage against each other. Using these probabilities, we can calculate the *naive* conditional probabilities that country i will win a war:

$$\hat{p}_i(A) = \hat{p}_i(B) = .55 \quad \text{and} \quad \hat{p}_i(C) = .4.$$

Thus, if $c_i > .05$, all types of country i are pessimistic. This fact is also displayed in Figure 5.

We are now ready to identify the equilibria of the game given in Figure 1 with this information structure. If $c_i > .05$ for $i = 1, 2$, then it is easy to check that this game has a perfect Bayesian equilibrium in which all types of both countries play action N. In this peaceful equilibrium, no type chooses to fight because its expected war payoff is equal to its naive conditional probability of victory minus its cost c_i and for every type this payoff is strictly less than .5. More interestingly, if $.05 < c_i < .15$ for $i = 1, 2$, then there also exists a perfect Bayesian equilibrium in which each country plays action F if its type is A or B and plays action N if its type is C. To see that this strategy profile is indeed an equilibrium, consider country 2. The choice of country 2 matters only if country 1 is choosing N, which

	A	B	C	
A	(.5,.5)	(.5,.5)	(.65,.35)	Pess.
B	(.5,.5)	(.5,.5)	(.65,.35)	Pess.
C	(.35,.65)	(.35,.65)	(.5,.5)	Pess.
	Pess.	Pess.	Pess.	

Figure 6: Type pairs at war and peace

only occurs if $t_1 = C$. Therefore, for type A of country 2, conditional on its action mattering, its expected payoff for war is $p_2(C, A) - c_2 = .65 - c_2$. As $c_2 < .15$, choosing F is superior to choosing N. The expected payoff for war for type B of country 2, conditional on its action mattering, is also $.65 - c_2$ so F is also optimal for for type B. Finally for type C of country 2, its expected payoff for war conditional on its action mattering is $p_2(C, C) - c_2 = .5 - c_2$. As $c_2 > 0$, this types of country 2 prefers to choose N rather than F. Turning now to the choice of country 1, note that country 1's choice matters only when country 2 is choosing action N. Given country 2's strategy, this occurs precisely when $t_2 = C$. Therefore, the analysis for country 1 is exactly symmetric to the analysis just described for country 2, which establishes that this strategy profile is an equilibrium.[6] It should also be noted that this is a strict perfect Bayesian equilibrium and therefore no type of either country is playing a weakly dominated action. Thus, this equilibrium is not ruled out by any standard refinement argument.

2.5 Example 2: Implications

We summarize the outcomes of the equilibrium we have just described in Figure 6. In this figure, the type pairs for which war occurs are shaded and the type pairs for which the peaceful settlement occurs are unshaded.

We again make two main observations about this example. The first observation is that war can occur in the absence of optimism on either side. Although this game has a peaceful equilibrium, it also has an equilibrium in which war occurs, even though no type of either country is optimistic. In this second equilibrium we see that for both sides, if the opponent's strategy is to fight unless they are the C type, then conditional on their choice mattering, each player when type A or B has a strict incentive to fight. Consequently there is an equilibrium with a high probability of war in this example even though no single actor of any type is optimistic given their private information.

[6] For this range of c_i, there also exists a perfect Bayesian equilibrium in which some types of both countries mix. This equilibrium thus also involves a positive probability of war.

The second observation is that the probability of war can be high, even in the absence of optimism. The ex ante probability of war in this equilibrium is 8/9. Comparing this information structure to that given in Example 1, we see that even though there is less optimism, there is an equilibrium with a higher probability of war. From these examples we cannot say mutual optimism is sufficient for war in this canonical model, we cannot say it is necessary, and we cannot claim that, in general, a higher likelihood of optimism is probabilistically associated with greater chances of war.

3 General Model

The example given in the previous section generates several suggestive observations about the connection between mutual optimism and war. But are these observations limited to our specific example or are they more broadly applicable? To address this question in this section we develop a general model of war in order to provide general results.

In what follows we consider two countries that face a potential conflict that can be settled either by force or by a negotiated settlement. We suppose that any negotiated settlement is efficient, but that war is inefficient. We also suppose that a war can be started by either side, unilaterally. To explore the role that private information plays in this choice, we assume that there is a set Ω of possible states of the world. Each possible state of the world, denoted ω, is a complete description of both countries' capabilities and prospects for war. As is standard, we suppose both countries share a *common prior* π on Ω and focus on how differences in information might lead to the choice of war.

In order to incorporate these states of the world into a conflict game, we present a general model of knowledge and connect it to the more familiar framework of Bayesian games. In this model of knowledge, we associate information or knowledge with the ability to distinguish between various states ω in Ω. We assume Nature initially draws the true state of the world according to the common prior π. Nature then provides information to players in the form of a signal about the true state of the world. The (deterministic) signal function of player i is denoted $t_i(\omega)$. In this setting, the type space of player i, T_i, is the range of the function $t_i(\omega)$. That is, the set of *types* of player i is just the set of all possible signals for player i. As Ω is assumed to be finite, the set T_i is also finite. The inverse image of the signal function, $t_i^{-1}(t_i^k)$, gives the set of states that could give rise to type t_i^k. These inverse image sets are important in the following way. Let $P_i(\omega) = \{\omega' \mid t_i(\omega) = t_i(\omega')\}$. We call $P_i(\omega)$ a *possibility correspondence*. For each $\omega \in \Omega$, $P_i(\omega)$ is interpreted as the collection of states that individual i thinks are possible when the true state is ω. This is one example of an *event*, which are naturally defined as subsets of Ω. A possibility correspondence $P_i(\omega)$ for Ω is *partitional* if there is a partition of Ω such that for any $\omega \in \Omega$ the set $P_i(\omega)$ is the element of the partition that contains ω. As shown by Rubinstein (1998) and others, a fully rational player must have a partitional possibility correspondence.

We now turn to incorporating this model of knowledge into a general model of war. Define two functions, $p_1(\omega)$ and $p_2(\omega)$, that specify the probability that country 1 and 2 will win a war, given the true state of the world ω. Of course, $p_1(\omega) + p_2(\omega) = 1$ and $0 \leq p_i(\omega) \leq 1$ for all values $\omega \in \Omega$. Consider an arbitrary event E. If a country knows an event $E \subseteq \Omega$ has occurred, it can combine this information with the prior π via Bayes' Rule to form a posterior belief about the value of p_i as follows:

$$\mathrm{E}[p_i|E] = \frac{\sum_{\omega \in E} p_i(\omega)\pi(\omega)}{\sum_{\omega \in E} \pi(\omega)} \tag{1}$$

From this expression, it is easy to verify that if $\mathrm{E}[p_i|E'] \geq x$ and $\mathrm{E}[p_i|E''] \geq x$ for disjoint sets of states E' and E'', then $\mathrm{E}[p_i|E' \cup E''] \geq x$. This result is known as the Sure Thing Principle (Savage 1954).

We normalize the utility of countries to be 1 for victory in war and 0 for defeat, and we suppose there is a cost $c_i(\omega) > 0$ of fighting a war for country i. Thus, in the event of war at state ω, the expected utility of country i is $p_i(\omega) - c_i(\omega)$. Similarly, it is possible that the potential negotiated settlement will depend on the private information of the two sides. Therefore, we define two additional functions, $r_1(\omega)$ and $r_2(\omega)$, that specify the bargaining outcome when the true state of the world is ω. These $r(\omega)$ functions could be generated from an axiomatic bargaining solution such as the Nash bargaining solution, the settlement of an incentive compatible mechanism, or any other function that generates a settlement for each state of the world.

Since bargaining is efficient, we assume that $r_1(\omega) + r_2(\omega) = 1$ for all values $\omega \in \Omega$. Given a true state ω, a country can combine its knowledge of $P_i(\omega)$ with the prior π via Bayes' Rule (equation 1) to construct its individual belief about the probability it will win, $\hat{p}_i(\omega) = \mathrm{E}[p_i|P_i(\omega)]$, the cost of fighting $\hat{c}_i(\omega) = \mathrm{E}[c_i|P_i(\omega)]$, and its expected payoff from bargaining, $\hat{r}_i(\omega) = \mathrm{E}[r_i|P_i(\omega)]$. In this setting, we say that country i is *optimistic* at ω if $\hat{p}_i(\omega) - \hat{c}_i(\omega) > \hat{r}_i(\omega)$. If exactly one country is optimistic at ω, then we say *unilateral optimism* occurs at ω; if both sides are optimistic at ω, then we say *mutual optimism* occurs at ω.

We end this section by describing the class of games that we analyze. Let the set of actions for player i in some two-player strategic form game be given by the set A_i. The result of the choice of actions for the two sides will be either war or a peaceful settlement. We assume that war is a *unilateral act*, so that either side can start a war. Formally, war is a unilateral act if, for each i, there is an action $\bar{a}_i \in A_i$ such that whatever action is chosen by the opponent, the outcome is war. To avoid redundancy, we assume that the action $\bar{a}_i \in A_i$ is the unique action with this property and, to avoid triviality, we assume that there is some action profile that results in a peaceful settlement, as well.

Finally, we define a pure strategy $s_i \in S_i$ as a function $s_i : \Omega \to A_i$ with the restriction that

$$P_i(\omega) = P_i(\omega') \quad \Rightarrow \quad s_i(\omega) = s_i(\omega').$$

This condition states that if a country cannot distinguish state ω from state ω', then its action must be the same in both states. For a given strategy profile (s_1, s_2), if there is a positive probability that the war outcome results from the play of this strategy profile, we say that (s_1, s_2) is a strategy profile in which *war occurs*. Since we have specified a strategic form game with incomplete information, the appropriate solution concept is Bayesian-Nash equilibrium. Note that under the assumption that war is a unilateral act, a pure strategy Bayesian-Nash equilibrium always exists, namely the strategy profile in which every type of country 1 chooses action $\bar{a}_1$ and every type of country 2 chooses action $\bar{a}_2$.

4 General Results

In this section we present two general results that apply to the broad class of games defined in the previous section. We first show that the existence of unilateral optimism precludes peace. We then give a result that show that mutual optimism is never necessary for war. Finally, we discuss how these results extend to cases in which actors are not fully rational in their decision makers.

Throughout this section, let G denote an arbitrary strategic form game of incomplete information that satisfies our assumptions on the information structure, payoffs, and strategies give in the previous section.

4.1 Unilateral Optimism

Our first result states that if at least one type of one country is optimistic, then there is a positive probability of war in equilibrium. That is, the possibility of unilateral optimism precludes peace. In addition, the converse of this statement is also true. If neither country has an optimistic type, then there exists a peaceful equilibrium.

Theorem 1 *Let G denote an arbitrary strategic form game of incomplete information in which war is a unilateral act. Then there is a positive probability of war in every pure strategy Bayesian-Nash equilibrium of G if and only if there is a state ω and a country i that is optimistic at ω.*

Proof: We begin by showing that if there is a state ω and a country i that is optimistic at ω, then in every pure strategy Bayesian-Nash equilibrium of G there is a positive probability of war. For a proof by contradiction, suppose that there is a game G with a state ω and a country i that is optimistic at ω and a pure strategy Bayesian-Nash equilibrium with zero probability of war. This means that type $t_i(\omega)$ of country i is not choosing action $\bar{a}_i$ in equilibrium and, moreover, this type's equilibrium payoff is $\hat{r}_i(\omega)$. If this type deviates to action $\bar{a}_i$, however, its payoff is $\hat{p}_i(\omega)-\hat{c}_i(\omega)$. Since country i is optimistic at ω, $\hat{p}_i(\omega)-\hat{c}_i(\omega) > \hat{r}_i(\omega)$ and therefore this deviation is profitable. This contradicts the existence of such an equilibrium.

For the reverse direction, we suppose that there is no state ω for which either side is optimistic and show the existence of a peaceful equilibrium. To do so, fix a strategy profile that gives the peaceful settlement at every state $\omega \in \Omega$. This is possible because, by assumption, there exists an action profile that results in the peaceful settlement. For this strategy profile, the expected payoff of type $t_i(\omega)$ of country i is $\hat{r}_i(\omega)$. Deviating to some other action will result in a peaceful settlement with probability one, war with probability one, or both outcomes with some positive probability. Thus, the payoff to deviating of type $t_i(\omega)$ of country i is a convex combination of $\hat{p}_i(\omega) - \hat{c}_i(\omega)$ and $\hat{r}_i(\omega)$. But because no type is optimistic, we have $\hat{p}_i(\omega) - \hat{c}_i(\omega) \leq \hat{r}_i(\omega)$. Therefore this is not a profitable deviation. Thus, such a strategy profile is indeed a peaceful equilibrium. ∎

This theorem states that the existence of optimism on the part of a single country is enough to ensure that war occurs in equilibrium. The logic is simple: in a completely peaceful equilibrium, an optimistic type has an incentive to fight. Importantly, optimism on only one side is enough for this result; it does not require mutual optimism. This fact makes the important point that it is not mutual optimism that drives the occurrence of war, instead one-sided optimism is enough. Of course, mutual optimism can be present when war occurs, but our result points out that it is not a requirement for war. Moreover, if both sides *always* lack optimism, then there exists a peaceful equilibrium.

It may be tempting to conclude from Theorem 1 that although the mutual optimism explanation is unsatisfactory, we are replacing it with an unsurprising result that countries fight when they think they are better off by fighting and they choose not to fight when they don't think this. However, the truth turns out to be significantly more subtle than this. First, as we demonstrated in Example 1 in Section 2, it is not the case that all optimistic types end up fighting. It is possible that a type who initially thinks fighting is better can rationally choose not to fight based on the strategic inference about what is actually true in the states where its choice matters. Second, Example 2 in Section 2 illustrates that it is not the case the optimism is needed for war to occur. To be clear, in this example there exists a peaceful equilibrium, as required by Theorem 1, but there also exists an equilibrium in which war occurs, even though no type of either country is optimistic. Again, the strategic inference that a side that is initially not optimistic makes can lead it to rationally choose to fight. This reinforces the point that mutual optimism is not always necessary for war by showing that even unilateral optimism is not always necessary for war.

4.2 Mutual Optimism

Our second result builds on this observation in another way. It proves that mutual optimism is not necessary in the entire class of games with unilateral war and not just in specific examples. That is, there are *no* examples of games with unilateral war in which war occurs *only* when mutual optimism holds.

Theorem 2 *Let G denote an arbitrary strategic form game of incomplete information in which war is a unilateral act. In every pure strategy Bayesian-Nash equilibrium of G in which war occurs, there is a state ω at which war occurs but mutual optimism does not hold.*

Proof: We begin by supposing that the strategy profile (s_1^*, s_2^*) is a Bayesian-Nash equilibrium in which war occurs. Denote the set of states for which the outcome of the game is war by W and denote the set of states for which the outcome is a peaceful settlement by T. As we are considering pure strategies, these two sets form a partition of Ω. Consider a state $\omega' \in T$. As each player can impose war by playing $\bar{a}_i$, and this deviation changes the payoff to player i only if war would not have occurred anyway, equilibrium requires

$$\begin{aligned} E[r_i(\omega) \mid P_i(\omega') \cap T] &\geq E[p_i(\omega) - c_i(\omega) \mid P_i(\omega') \cap T] \\ E[p_i(\omega) - c_i(\omega) - r_i(\omega) \mid P_i(\omega') \cap T] &\leq 0 \end{aligned} \tag{2}$$

for every $\omega' \in T$.

Now define the events

$$O_i = \{\omega \in \Omega \mid \hat{p}_i(\omega) - \hat{c}_i(\omega) > \hat{r}_i(\omega)\}$$

for $i = 1, 2$. To prove the theorem, suppose that the conclusion is false. That is, suppose that in every state that war occurs, mutual optimism also occurs. Formally, this requirement is that $W \subseteq O_1 \cap O_2$. Now, take an arbitrary $\omega \in W$. Because $\omega \in O_i$ for $i = 1, 2$, it follows that

$$E[p_i(\omega) - c_i(\omega) - r_i(\omega) \mid P_i(\omega)] > 0, \quad i = 1, 2. \tag{3}$$

We claim that for an arbitrary $\omega \in W$,

$$E[p_i(\omega) - c_i(\omega) - r_i(\omega) \mid P_i(\omega) \cap W] > 0, \quad i = 1, 2. \tag{4}$$

If $P_i(\omega) \cap T$ is empty, then $P_i(\omega) \cap W = P_i(\omega)$ and the claim follows from inequality (3). If $P_i(\omega) \cap T$ is nonempty, then there is some $\omega' \in P_i(\omega)$ such that $\omega' \in T$. Therefore, by inequality (2),

$$E[p_i(\omega) - c_i(\omega) - r_i(\omega) \mid P_i(\omega) \cap T] \leq 0.$$

As W and T form a partition of Ω, this implies that inequality (4) must hold because otherwise the Sure Thing Principle would generate a contradiction with inequality (3). Thus, in either case, inequality (4) holds.

As the correspondence P_i is partitional, we can define a set of states D_i^* with $D_i^* \subseteq W$ such that the sets $\{P_i(\omega)\}_{\omega \in D_i^*}$ are disjoint and

$$\bigcup_{\hat{\omega} \in D_i^*} [P_i(\hat{\omega}) \cap W] = \bigcup_{\hat{\omega} \in W} [P_i(\hat{\omega}) \cap W].$$

Since $D_i^* \subseteq W$, we have from inequality (4) that

$$E[p_i(\omega) - c_i(\omega) - r_i(\omega) \mid P_i(\omega) \cap W] > 0$$

for every $\hat{\omega} \in D_i^*$. As this holds for each disjoint set $P_1(\hat{\omega})$, then by the Sure Thing Principle the same conditional expectation inequality holds over the union of these disjoint sets. Therefore,

$$E[p_i(\omega) - c_i(\omega) - r_i(\omega) \mid \bigcup_{\hat{\omega} \in D_i^*} [P_i(\omega) \cap W]\,] > 0$$
$$E[p_i(\omega) - c_i(\omega) - r_i(\omega) \mid \bigcup_{\hat{\omega} \in W} [P_i(\omega) \cap W]\,] > 0.$$

As $\omega \in P_i(\omega)$ for every ω, it follows that

$$\bigcup_{\hat{\omega} \in W} [P_i(\omega) \cap W] = W.$$

We conclude that, for $i = 1, 2$,

$$E[p_i(\omega) - c_i(\omega) - r_i(\omega) \mid W] > 0.$$

From this, it follows that

$$E[p_1(\omega) - c_1(\omega) - r_1(\omega) \mid W] + E[p_2(\omega) - c_2(\omega) - r_2(\omega) \mid W] > 0$$
$$E[p_1(\omega) + p_2(\omega) \mid W] - E[r_1(\omega) + r_2(\omega) \mid W] > E[c_1(\omega) + c_2(\omega) \mid W]$$

As $p_1(\omega)+p_2(\omega) = 1$ and $r_1(\omega)+r_2(\omega) = 1$ for all $\omega \in \Omega$, it follows that $E[p_1(\omega)+p_2(\omega) \mid W] = 1$ and $E[r_1(\omega) + r_2(\omega) \mid W] = 1$. Thus we have

$$0 > E[c_1(\omega) + c_2(\omega) \mid W]$$

But this contradicts the fact that $c_i(\omega) > 0$ for all $\omega \in \Omega$, which proves the result. ∎

The theorem shows that there cannot be an equilibrium to a game in which countries have the ability to unilaterally start a war where mutual optimism is a necessary condition for costly conflict. This does not mean that mutual optimism and war cannot occur together in equilibrium, but rather that it is never necessary. Any game where war occurs in equilibrium must have realizations of the state of the world—in the particular equilibrium—where war occurs and there is no mutual optimism. Put more simply, Theorem 2 establishes that mutual optimism is never a necessary condition for war by showing that if war happens with mutual optimism, it must also occur without mutual optimism.

4.3 Bounded Rationality

In earlier work, Fey & Ramsay (2007) show that their arguments about the logical connection between mutual optimism and war continue to hold even when actors are not fully rational in their decision making. Thus, while Theorem 2 is true for a broad class of games in which the decision-makers rationally process information, here we pause to consider whether this result depends on strictly rational learning. We find that even if actors' information processing suffers from cognitive biases, the link between mutual optimism and war is still quite weak. In particular, even if both players ignore "bad news" or are inattentive, then mutual optimism is never necessary for war.

Consider an information structure that captures several possible kinds of information processing errors found in the psychological international relations literature (Jervis, Lebow & Stein 1985, Jervis 1976). This information structure is defined as follows.

Definition 1 *Let P_i be a possibility correspondence for individual i. We say P_i is*

1. nondeluded *if, for all $\omega \in \Omega$, $\omega \in P_i(\omega)$, and*
2. nested *if for all $\omega, \omega' \in \Omega$, either $P_i(\omega) \cap P_i(\omega') = \emptyset$, or $P_i(\omega) \subseteq P_i(\omega')$, or $P_i(\omega') \subseteq P_i(\omega)$.*

An individual with a possibility correspondence that is nondeluded and nested may "ignore" or "throw out" information that would be known to a fully rational Bayesian. This formalization is consistent with many forms of information processing bias, because it is agnostic to the reason information is ignored. Individuals could fail to learn in some states because acquiring information is costly, because they are inattentive, or because they would rather not think about the implications of the information in front of them (e.g., White 1968). For example, a ruler who would recognize an imminent victory, but never realize they facing a certain defeat in war would have such a correspondence.

We now establish that even with actors that do not process information in a fully rational way, mutual optimism is still never necessary for war.

Theorem 3 *Let G denote an arbitrary strategic form game of incomplete information in which war is a unilateral act, countries have a common prior, and P_i is nondeluded and* *nested for $i = 1, 2$. In every pure strategy Bayesian-Nash equilibrium of G in which war occurs, there is a state ω at which war occurs but mutual optimism does not hold.*

Proof: The proof of this theorem is very similar to the proof of Theorem 2. As in that proof, fix a pure strategy equilibrium (s_1^*, s_2^*) in which war occurs and let the set of states for which the outcome of the game is war be W and the set of states for which the outcome is a peaceful settlement be T. Using the exact same argument as in the proof of Theorem 2, we can establish that equation (4) continues to hold. That is, for an arbitrary $\omega \in W$,

$$E[p_i(\omega) - c_i(\omega) - r_i(\omega) \mid P_i(\omega) \cap W] > 0, \quad i = 1, 2. \tag{5}$$

Because information partitions can be nested, a given state of the world could belong to multiple information partitions. So let $M_i(\omega)$ be the largest set (with respect to set inclusion) of the collection of sets $\{P_i(\omega') \mid \omega \in P_i(\omega')$. By nestedness, $M_i(\omega)$ is well-defined for all $\omega \in \Omega$. Moreover, by non-deluded and nestedness, for all $\omega, \omega' \in \Omega$, either $M_i(\omega) = M_i(\omega')$ or $M_i(\omega) \cap M_i(\omega') = \emptyset$. Therefore the $M_i(\omega)$ sets form a partition of Ω. Enumerate the sets that make up this partition for i as $M_i^1, M_i^2, \ldots, M_i^K$. For each set M_i^k such that $M_i^k \cap W \neq \emptyset$, let

$$\bar{P}_i^k = \bigcup_{\omega \in M_i^k \cap W} P_i(\omega) \cap W$$

By nestedness, $\bar{P}_i^k = P_i(\omega') \cap W$ for some $\omega' \in W$. Therefore, by equation (5), we have

$$E[p_i(\omega) - c_i(\omega) - r_i(\omega) \mid \bar{P}_i^k] > 0, \quad i = 1, 2.$$

Moreover, by non-deluded, $\bar{P}_i^k = M_i^k \cap W$. As the M_i sets form a partition of Ω, the $\bar{P}_i^k$ sets just defined form a partition of W. Thus we can then write W as the union of disjoint sets $\bar{P}_i^k$, defined by some collection of states $\hat{D}^*$ all contained in W, i.e., $\hat{D}^* \subseteq W$. The result then follows as in Theorem 2. ∎

Theorem 3 show that for some plausible types of "boundedly rational" actors, mutual optimism cannot be necessary for war. Or, in other words, if war and mutual optimism occur simultaneously at some state in an equilibrium in G, then there must be some another state where there is war and no mutual optimism. Therefore, the mutual optimism result in Theorem 2 is not fragile. Clearly, some departure from rational Bayesian learning is acceptable and consistent with our results. In particular, if decision-makers sometimes ignore unpleasant information or behave as if they have imperfect memory, then our result survives.

5 Conclusion

Uncertainty about the military capabilities of opponents has long been argued to be an important causal explanation for war between countries. Here we have taken up the question of how strategic reasoning and this uncertainty interact to better understand strategic decision-making leading up to war. In particular, we focus on the force of strategic inference when war is the result of a unilateral choice by a single country to forego a known efficient settlement.

Our summary finding is that even in very simple circumstances there is a limited logical link between a decision-makers beliefs on the eve of war and war onset. A decision to go to war need not accompany optimism on the decision-makers behalf and a pair of leaders who both think that there understanding of the military balance makes war look better than the expected peaceful settlement may still *agree* to settle. The empirical implication of these findings is that we may see war occur when decision-makers are both optimistic, when only one side is optimistic, or even when no one is optimistic. But this does not mean that there is nothing to say about uncertainty about victory and war.

In general, we can say instead that there is war with positive probability in every equilibrium if and only if there is there is a state where some country is naively optimistic when they know only their type. We can also say that there are no situations in which, in equilibrium, war occurs only at states of the world where decision-makers are both optimistic. And these results are quite robust. Therefore, in an important way, the empirical relationship between mutual optimism and war that we see in some cases may be just coincidental. Unilateral optimism, however, is a more important marker for war. While optimistic states may or may not fight in any realized situation, the possibility of unilateral optimism alone implies that the *ex ante* probability of war is always positive in any pure strategy equilibrium.

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Reviewer's Appendix

Here we show that there is a unique perfect Bayesian equilibrium to Example 1 in Section 2. The set of actions of both countries is $A_i = \{F, N\}$ and the set of types of each country is $T_i = \{A, B, C\}$. Thus a (mixed) strategy for country i gives the probability that each type t_i plays N, which we denote by $\sigma_i(t_i)$. As the game in the example is an extensive form game, we must also define the beliefs of country 2. Let $\mu_k = P[t_1 = k \mid a_1 = N]$ for $k \in \{A, B, C\}$ denote such a belief. Using these beliefs, it is sequentially rational for type t_2 of country 2 to choose F if

$$\begin{gathered}
Eu_2(F \mid t_2, \mu) \geq Eu_2(N \mid t_2, \mu) \\
\mu_A p_2(A, t_2) + \mu_B p_2(B, t_2) + \mu_C p_2(C, t_2) - c_2 \geq 1/2 \\
\mu_A p_2(A, t_2) + \mu_B p_2(B, t_2) + \mu_C p_2(C, t_2) \geq 1/2 + c_2.
\end{gathered}$$

Turning now to the choice of country 1, it is sequentially rational for type t_1 of country 1 to choose F if

$$\begin{gathered}
Eu_1(F \mid t_1, \sigma_2) \geq Eu_1(N \mid t_1, \sigma_2) \\
(1/3)[p_1(t_1, A) + p_1(t_1, B) + p_1(t_1, C)] - c_2 \geq (1/3)[(1 - \sigma_2(A))(p_1(t_1, A) - c_1) + \sigma_2(A)(1/2) \\
+(1 - \sigma_2(B))(p_1(t_1, B) - c_1) + \sigma_2(B)(1/2) + (1 - \sigma_2(C))(p_1(t_1, C) - c_1) + \sigma_2(C)(1/2)] \\
\sigma_2(A)(p_1(t_1, A) - c_1 - 1/2) + \sigma_2(B)(p_1(t_1, B) - c_1 - 1/2) + \sigma_2(C)(p_1(t_1, C) - c_1 - 1/2) \geq 0.
\end{gathered}$$

The fact that each term in this expression is weighted by the $\sigma_2(t_2)$ reflects the fact that country 1's choice of action only matters if country 2 is choosing N.

We must show that the only perfect Bayesian equilibrium to this game is one in which $\sigma_i(A) = 0$ and $\sigma_i(B) = \sigma_i(C) = 1$ for $i = 1, 2$. To begin, consider a type C of country 2. It is easy to see from the above condition that there is no belief μ that makes fighting sequentially rational. Therefore $\sigma_2^*(C) = 1$ in any perfect Bayesian equilibrium. Using this, we see that because $p_1(C, t_2) \leq .5$ for all types t_2, type C of country 1 will never play F. Thus, $\sigma_1^*(C) = 1$ in any perfect Bayesian equilibrium. Now we turn to type A of country 1. For this type, F is strictly preferred to N if

$$\sigma_2(A)(-c_1) + \sigma_2(B)(-.2 - c_1) + (.4 - c_1) > 0.$$

But note that

$$\sigma_2(A)(-c_1) + \sigma_2(B)(-.2 - c_1) + (.4 - c_1) \geq (-c_1) + (-.2 - c_1) + (.4 - c_1) = .2 - 3c_1 > 0,$$

where the last inequality follows from $c_1 < 1/15$. This implies that $\sigma_1^*(A) = 0$ in any perfect Bayesian equilibrium. A similar argument establishes that $\sigma_2^*(A) = 0$ in any perfect Bayesian equilibrium. It then follows easily that $\sigma_1^*(B) = \sigma_2^*(B) = 1$. Thus, the equilibrium in which only the A type of each country fights is the unique perfect Bayesian equilibrium to this game.